

Maximum mass of neutron stars with a quark core

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Abstract. Massive neutron stars (NS) are expected to possess a quark core. While the hadronic side of the NS equation of state (EOS) can be considered well established, the quark side is quite uncertain. While calculating the EOS of hadronic matter we have used the Brueckner-Bethe-Goldstone formalism with realistic two-body and three-body forces, as well as a relativistic mean field model. For quark matter we employ the MIT bag model constraining the bag constant by exploiting the recent experimental results obtained at CERN on the formation of a quark-gluon plasma. We calculate the structure of NS interiors with the EOS comprising both phases, and we find that the NS maximum masses fall in a relatively narrow interval, $1.45 M_{\odot} \leq M_{\text{max}} \leq 1.65 M_{\odot}$, near the lower limit of the observational range.

An ongoing active research area, both theoretical and experimental, concerns the properties of matter under extreme conditions of density and temperature, and the determination of the EOS associated with it. Its knowledge is of key importance for building models of neutron stars (NS's) [1]. The observed NS masses are typically $\approx (1 - 2)M_\odot$ (where M_\odot is the mass of the sun, $M_\odot = 1.99 \times 10^{33}\text{g}$), and the radius is of the order of 10 km. The matter in the core possesses densities ranging from a few times ρ_0 ($\approx 0.17\text{ fm}^{-3}$, the normal nuclear matter density) to one order of magnitude higher. Therefore, a detailed knowledge of the EOS is required for densities $\rho \gg \rho_0$, where a description of matter only in terms of nucleons and leptons may be inadequate. In fact, at densities $\rho \gg \rho_0$ several species of other particles, such as hyperons and Δ isobars, may appear, and meson condensations may take place; also, ultimately, at very high densities, nuclear matter is expected to undergo a transition to a quark-gluon plasma [2]. However, the exact value of the transition density to quark matter is unknown and still a matter of recent debate.

In this letter, we propose to constrain the maximum mass of neutron stars taking into account the phase transition from hadronic matter to quark matter inside the neutron star. For this purpose, we describe the hadron phase of matter by using two different equations of state, *i.e.* a microscopic EOS obtained in the Brueckner -Bethe -Goldstone (BBG) theory [3], and a more phenomenological relativistic mean field model [4]. The deconfined quark phase is treated within the popular MIT bag model [5]. The bag constant, B , which is a parameter of the bag model, is constrained to be compatible with the recent experimental results obtained at CERN on the formation of a quark-gluon plasma [6], recently confirmed by RHIC preliminary results [7]. This statement requires some clarification. In general, it is not obvious if the informations on the nuclear EOS from high energy heavy ion collisions can be related to the physics of neutron stars interior. The possible quark-gluon plasma produced in heavy ion collision is expected to be characterized by small baryon density and high temperature, while the possible quark phase in neutron stars appears at high baryon density and low temperature. However, if one adopts for the hadronic phase a non-interacting gas model of nucleons, antinucleons and pions, the original MIT bag model predicts that the deconfined phase occurs at an almost constant value of the quark-gluon energy density, irrespective of the thermodynamical conditions of the system [8]. For this reason, it is popular to draw the transition line between the hadronic and quark phase at a constant value of the energy

density, which was estimated to fall in the interval between 0.5 and 2 GeV fm⁻³ [9]. This is consistent with the value of about 1 GeV fm⁻³ reported by CERN experiments.

In this exploratory work we will assume that this is still valid, at least approximately, when correlations in the hadron phase are present. We will then study the predictions that one can draw from this hypothesis on neutron star structure. Any observational data on neutron stars in disagreement with these predictions would give an indication on the accuracy of this assumption. Indeed, the hadron phase EOS can be considered well established. The main uncertainty is contained in the quark phase EOS, since it can be currently described only by phenomenological models which contain few adjustable parameters. In the case of the MIT bag model, which is adopted in this work, the parameters are fixed to be compatible with the CERN data, according to the hypothesis of a constant energy density along the transition line. In practice, this means that all our calculations can be limited to zero temperature.

We start with the description of the hadronic phase. It has been shown that the non-relativistic BBG expansion is well convergent [10], and the Brueckner-Hartree-Fock (BHF) level of approximation is accurate in the density range relevant for neutron stars. In the calculations reported here we have used the Paris potential [11] as the two-nucleon interaction and the Urbana model as three-body force [12]. This allows the correct reproduction of the empirical nuclear matter saturation point ρ_0 [13]. Recently the above procedure has been extended to the case of asymmetric nuclear matter including hyperons [14, 15] by utilizing hyperon-nucleon potentials that are fitted to the existing scattering data.

To complete our analysis, we will consider also a hadronic EOS derived from relativistic mean field model (RMF) [16]. The BHF and the RMF EOS are both shown in Fig.1. The parameters of the RMF model have been taken in such a way that the compressibility at saturation is around 260 MeV, the same as in BHF calculations and close to estimates from monopole oscillations in nuclei [17]. The symmetry energy is also quite similar for the two EOS, about 30 MeV at saturation.

For the deconfined quark phase, within the MIT bag model [5], the total energy density is the sum of a non-perturbative energy shift B , the bag constant, and the kinetic energy for non-interacting massive quarks of flavors f with mass m_f and Fermi momentum $k_F^{(f)} [= (\pi^2 \rho_f)^{1/3}]$, with ρ_f as the quarks'

density of flavour f]

$$\frac{E}{V} = B + \sum_f \frac{3m_f^4}{8\pi^2} \left[x_f \sqrt{x_f^2 + 1} (2x_f^2 + 1) - \sinh^{-1} x_f \right], \quad (1)$$

where $x_f = k_F^{(f)}/m_f$. We consider in this work massless u and d quarks, whereas the s quark mass is taken equal to 150 MeV. The bag constant B can be interpreted as the difference between the energy densities of the perturbative vacuum and the physical vacuum. Inclusion of perturbative interaction among quarks introduces additional terms in the thermodynamic potential [18] and hence in the number density and the energy density; however, when taken into account in the first order of the strong coupling constant, these terms do not change our results appreciably. Therefore, in order to calculate the EOS for quark matter we restrict to Eq. (1). In the original MIT bag model the bag constant has the value $B \approx 55 \text{ MeV fm}^{-3}$, which is quite small when compared with the ones ($\approx 210 \text{ MeV fm}^{-3}$) estimated from lattice calculations [19]. In this sense B can be considered as a free parameter.

We try to determine a range of possible values for B by exploiting the experimental data obtained at the CERN SPS, where several experiments using high-energy beams of Pb nuclei reported (indirect) evidence for the formation of a quark-gluon plasma [6]. The resulting picture is the following: during the early stages of the heavy-ion collision, a very hot and dense state (fireball) is formed whose energy materializes in the form of quarks and gluons strongly interacting with each other, exhibiting features consistent with expectations from a plasma of deconfined quarks and gluons [20]. Subsequently, the “plasma” cools down and becomes more dilute up to the point where, at an energy density of about 1 GeV fm^{-3} and temperature $T \approx 170 \text{ MeV}$, the quarks and gluons hadronize. The expansion is fast enough so that no mixed hadron-quark equilibrium phase is expected to occur, and no weak process can play a role. According to the analysis of those experiments, the quark-hadron transition takes place at about seven times normal nuclear matter energy density ($\epsilon_0 \approx 156 \text{ MeV fm}^{-3}$). In the MIT bag model, the structure of the QCD phase diagram in the chemical potential and temperature plane is determined by only one parameter, B , although the phase diagram for the transition from nuclear matter to quark matter is schematic and not yet completely understood, particularly in the light of recent investigations on a color superconducting phase of quark matter [21]. As discussed above, in our analysis we assume that the transition to quark-gluon plasma is determined

by the value of the energy density only (for a given asymmetry). With this assumption and taking the hadron to quark matter transition energy density from the CERN experiments we estimate the value of B and its possible density dependence as given below.

First, we calculate the EOS for cold asymmetric nuclear matter characterized by a proton fraction $x_p = 0.4$ (the one for Pb nuclei accelerated at CERN-SPS energies) in the BHF formalism with two-body and three-body forces as described earlier. The result is shown by the solid line in Fig. 1a). Then we calculate the EOS for u and d quark matter using Eq. (1). We find that at very low baryon density the quark matter energy density is higher than that of nuclear matter, while with increasing baryon density the two energy densities become equal at a certain point [indicated in Fig. 1a) by the full dot)], and after that the nuclear matter energy density remains always higher. We identify this crossing point with the transition density from nuclear matter to quark matter. To be more precise, this crossing fixes the density interval where the phase transition takes place. In fact, according to the Gibb's construction, the crossing must be located at the center of the mixed phase region, if it is present. To be compatible with the experimental observation at the CERN-SPS, we require that this crossing point corresponds to an energy density of $E/V \approx 7\epsilon_0 \approx 1.1 \text{ GeV fm}^{-3}$. However, for no density independent value of B , the two EOS' cross each other, satisfying the above condition. Therefore, we try a density dependent B . In the literature there are attempts to understand the density dependence of B [22]; however, currently the results are highly model dependent and no definite picture has come out yet. Therefore, we attempt to provide effective parametrizations for this density dependence, trying to cover a wide range by considering some extreme choices. Our parametrizations are constructed in such a way that at asymptotic densities B has some finite value B_{as} . We have found $B_{as} = 50 \text{ MeV fm}^{-3}$ for the BHF case, but have verified that our results do not change appreciably by varying this value, since at large densities the quark matter EOS is dominated by the kinetic term on the RHS of Eq. (1). First, we use a Gaussian parametrization given as

$$B(\rho) = B_{as} + (B_0 - B_{as}) \exp \left[-\beta \left(\frac{\rho}{\rho_0} \right)^2 \right]. \quad (2)$$

The parameter β has been fixed by equating the quark matter energy density from Eq. (1) with the nucleonic one at the desired transition density $\rho_c = 0.98 \text{ fm}^{-3}$ (represented by the full dot in Fig. 1a)), i.e. $E/V(\rho_c) =$

1.1 GeV fm^{-3} . Therefore β will depend only on the free parameter $B_0 = B(\rho = 0)$. However, the exact value of B_0 is not very relevant for our purpose, since at low density the matter is in any case in the nucleonic phase. We attempt to cover the typical range by using the values $B_0 = 200 \text{ MeV fm}^{-3}$ and 400 MeV fm^{-3} , as shown in Fig. 1c). We also use another extreme, Woods-Saxon like, parametrization,

$$B(\rho) = B_{as} + (B_0 - B_{as}) \left[1 + \exp \left(\frac{\rho - \bar{\rho}}{\rho_d} \right) \right]^{-1}, \quad (3)$$

where B_0 and B_{as} have the same meaning as described before for Eq. (2) and $\bar{\rho}$ has been fixed in the same way as β for the previous parametrization. For $B_0 = 400 \text{ MeV fm}^{-3}$, we get $\bar{\rho} = 0.8 \text{ fm}^{-3}$ for $\rho_d = 0.03 \text{ fm}^{-3}$. With this parametrization B remains practically constant at a value B_0 up to a certain density and then drops to B_{as} almost like a step function, as shown by the long-dashed curve in Fig. 1c). It is an extreme parametrization in the sense that it will delay the onset of the quark phase in neutron star matter as much as possible. Both parametrizations Eqs. (2) and (3) yield the transition from nuclear matter to quark matter at the energy density compatible with the experiments.

The same procedure has been followed for the RMF EOS, see Figs. 1b) and 1d). In this case the parameter B_{as} is slightly smaller, about 38 MeV fm^{-3} . With these parametrizations of the density dependence of B we now consider the hadron-quark phase transition in neutron stars. We calculate in the BHF framework and in the RMF approach the EOS of a conventional neutron star as composed of a chemically equilibrated and charge neutral mixture of nucleons, leptons and hyperons. The result is shown by the solid lines in Fig. 2a) and 2b) respectively. The other curves (with the same notation as in Fig. 1) represent the EOS' for beta-stable and charge neutral quark matter. We determine the range of baryon density where both phases can coexist by following the construction from ref. [23]. In this procedure both hadron and quark phases are allowed to be charged, still preserving the total charge neutrality. Pressure is the same in the two phases to ensure mechanical stability, while the chemical potentials of the different species are related to each other to ensure chemical and beta stability. The resulting EOS for neutron star matter, according to the different bag parametrizations, is reported in Fig. 3, where the shaded area indicates the mixed phase region. A pure quark phase is present at densities above the shaded area and a pure

hadronic phase is present below it. The onset density of the mixed phase turns out to be slightly smaller than the density for hyperons formation in pure hadronic matter. Of course hyperons are still present in the hadron component of the mixed phase. For the Wood-Saxon parametrization of the bag constant the mixed phase persists up to high baryon density. As previously anticipated, this is in agreement with the delayed crossing of the energy density curves for the hadron and quark phases, as can be seen from Fig. 2. Finally, we solve the Tolman-Oppenheimer-Volkoff equations [1] for the mass of neutron stars with the EOS' of Figs. 3 as input. The calculated results, the NS mass vs. central density, for all cases are shown in Fig. 4a) and 4b). The EOS with nucleons, leptons and hyperons gives a maximum mass of neutron stars of about $1.26 M_{\odot}$ in the BHF case. In the case of the RMF model, the corresponding EOS produces values of the maximum mass close to $1.7 M_{\odot}$. It is commonly believed that the inclusion of the quark component should soften the NS matter EOS. This is indeed the case in the RMF model, as apparent in Fig. 4b). However the situation is reversed in the BHF case, where the EOS becomes, on the contrary, stiffer. Correspondingly, the inclusion of the quark component has the effect of increasing the maximum mass in the BHF case and of decreasing it in the RMF case. As a consequence, the calculated maximum masses fall in any case in a relatively narrow range, $1.45 M_{\odot} \leq M_{\text{max}} \leq 1.65 M_{\odot}$, slightly above the observational lower limit of $1.44 M_{\odot}$ [24].

As one can see from Fig. 4, the presence of a mixed phase produces a sort of plateau in the mass vs. central density relationship, which is a direct consequence of the smaller slope displayed by all EOS in the mixed phase region, see Fig. 3. In this region, however, the pressure is still increasing monotonically, despite the apparent smooth behaviour, and no unstable configuration can actually appear. We found that the appearance of this slow variation of the pressure is due to the density dependence of the bag constant, in particular the occurrence of the density derivative of the bag constant in the pressure and chemical potentials, as required by thermodynamic consistency. To illustrate this point we calculate the EOS for quark matter with a density independent value of $B = 90 \text{ MeV fm}^{-3}$, see Fig. 5, and the corresponding neutron star masses. The EOS is now quite smooth and the mass vs. central density shows no indication of a plateau. More details on this point will be given elsewhere [25].

Finally, it has to be pointed out that the maximum mass value, whether B is density dependent or not, is dominated by the quark EOS at densities

where the bag constant is much smaller than the quark kinetic energy. The constraint coming from heavy ion reactions, as discussed above, is relevant only to the extent that it restricts B at high density within a range of values, which are commonly used in the literature. This can be seen also from Fig. 5, where the (density independent) value of $B = 90\text{MeV}$ produces again a maximum value around 1.5 solar mass.

In conclusion, under our hypothesis, we found first that a density dependent B is necessary to understand the CERN-SPS findings on the phase transition from hadronic matter to quark matter. Then, taking into account this observation, we calculated NS maximum masses, using an EOS which combines reliable EOS's for hadronic matter and a bag model EOS for quark matter. The calculated maximum NS masses lie in a narrow range in spite of using very different parametrizations of the density dependence of B . Other recent calculations of neutron star properties employing various RMF nuclear EOS' together with either effective mass bag model [26] or Nambu-Jona-Lasinio model [27] EOS' for quark matter, also give maximum masses of only about $1.7 M_{\odot}$, even though not constrained to reproduce simultaneously the CERN-SPS data. The value of the maximum mass of neutron stars obtained according to our analysis appears robust with respect to the uncertainties of the nuclear EOS. Therefore, the experimental observation of a heavy ($M > 1.6 M_{\odot}$) neutron star, as claimed recently by some groups [28] ($M \approx 2.2 M_{\odot}$), if confirmed, would suggest mainly two possibilities. Either serious problems are present for the current theoretical modelling of the high-density phase of nuclear matter, or the working hypothesis that the transition to the deconfined phase occurs approximately at the same energy density, irrespective of the thermodynamical conditions, is substantially wrong. In both cases, one can expect a well defined hint on the high density nuclear matter EOS.

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Figure captions.

Figure 1: (a,b) The energy density E/V vs. the baryon density ρ for nuclear matter and quark matter of charge fraction $x_p = 0.4$. The dot indicates the common intersection of the curves. (c,d) Density dependence of the bag constant B (see text for details).

Figure 2: The energy density vs. baryon density for pure hadron matter (full lines) are reported for the BHF (left panel) and RMF (right panel) schemes, in comparison with the quark energy densities (broken lines) with different parametrizations of the bag constant.

Figure 3: Total EOS including both hadronic and quark components. Different prescriptions for the quark phase are considered, see the text and Figs. 1 and 2. For the hadron component the BHF (left panel) and the RMF (right panel) schemes are considered. In all cases the shaded region indicates the mixed phase MP, while HP and QP label the portion of the EOS where pure hadron and pure quark phases, respectively, are present.

Figure 4: The gravitational mass of neutron stars vs. the central density for the EOS's shown in Fig.3.

Figure 5: In the left panel is shown the EOS for neutron star matter (dashed lines labeled by HP + QP) for a density independent value of the bag constant $B = 90 \text{ MeV fm}^{-3}$, with BHF (a) and RMF (c) hadron equations of state. The shaded areas indicate the mixed phase region. The corresponding masses vs. central density are shown on the right panels. In all cases the thin and thick lines correspond to the results obtained for pure quark and pure hadron EOS respectively.

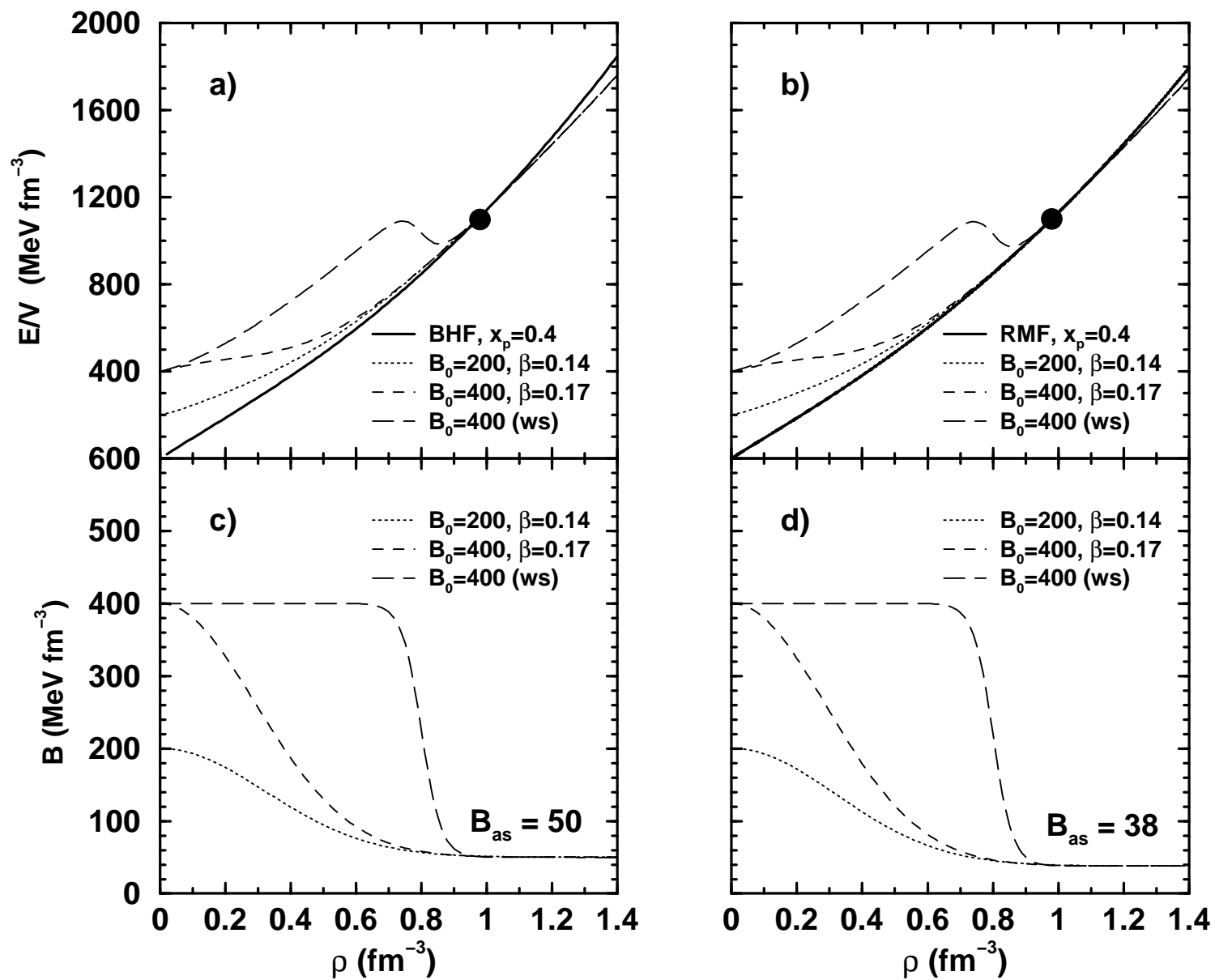


Fig.1

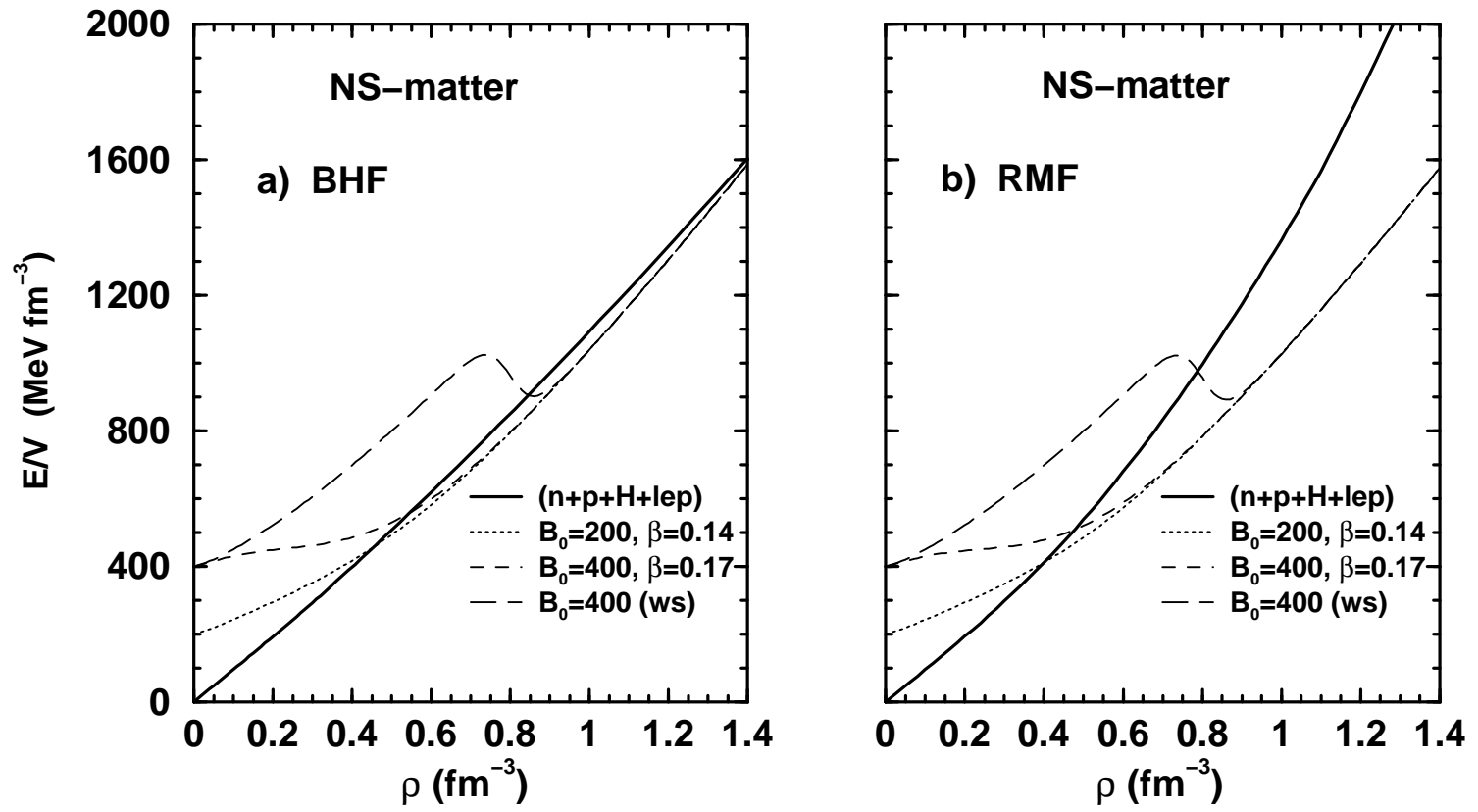


Fig.2

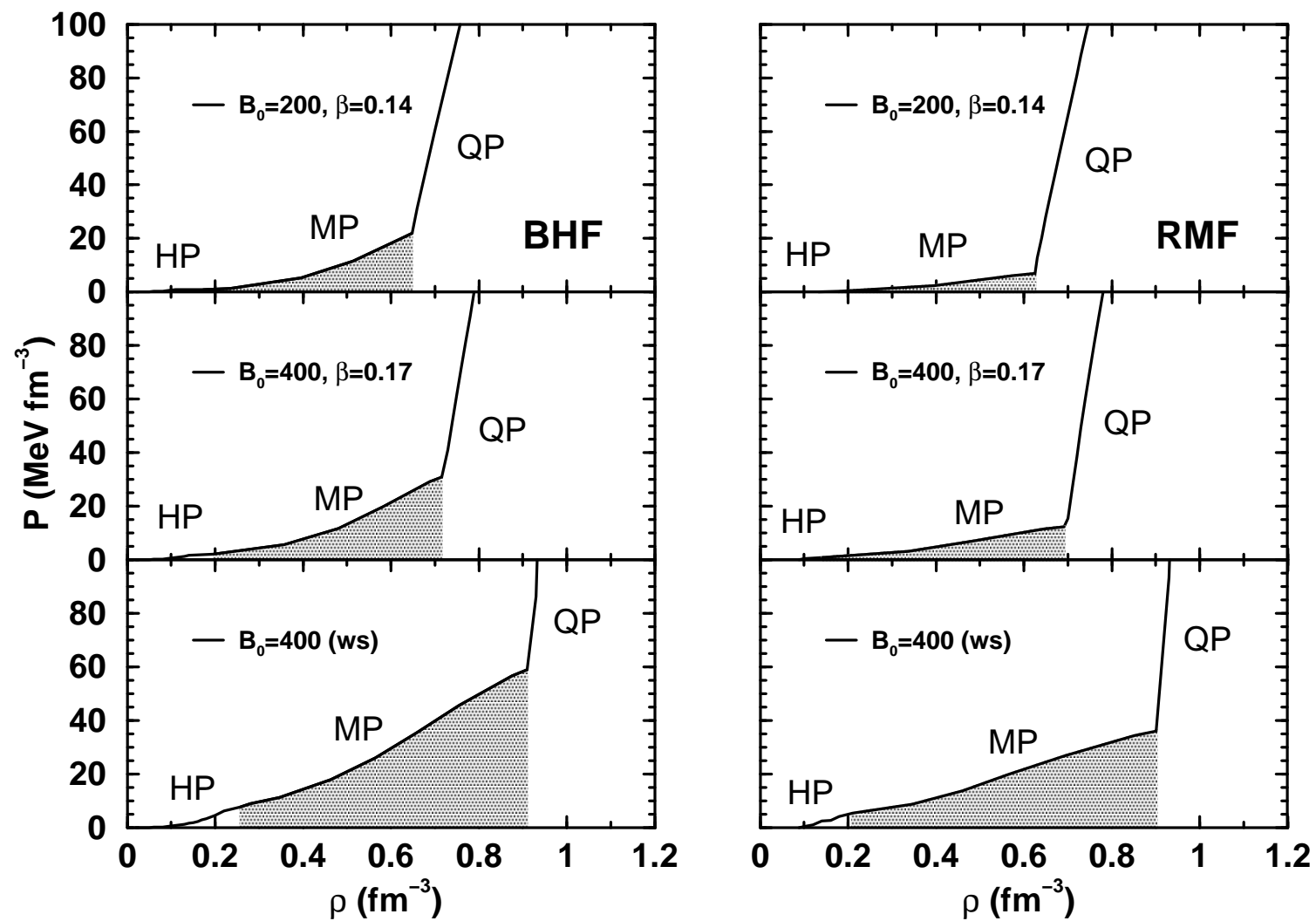


Fig.3

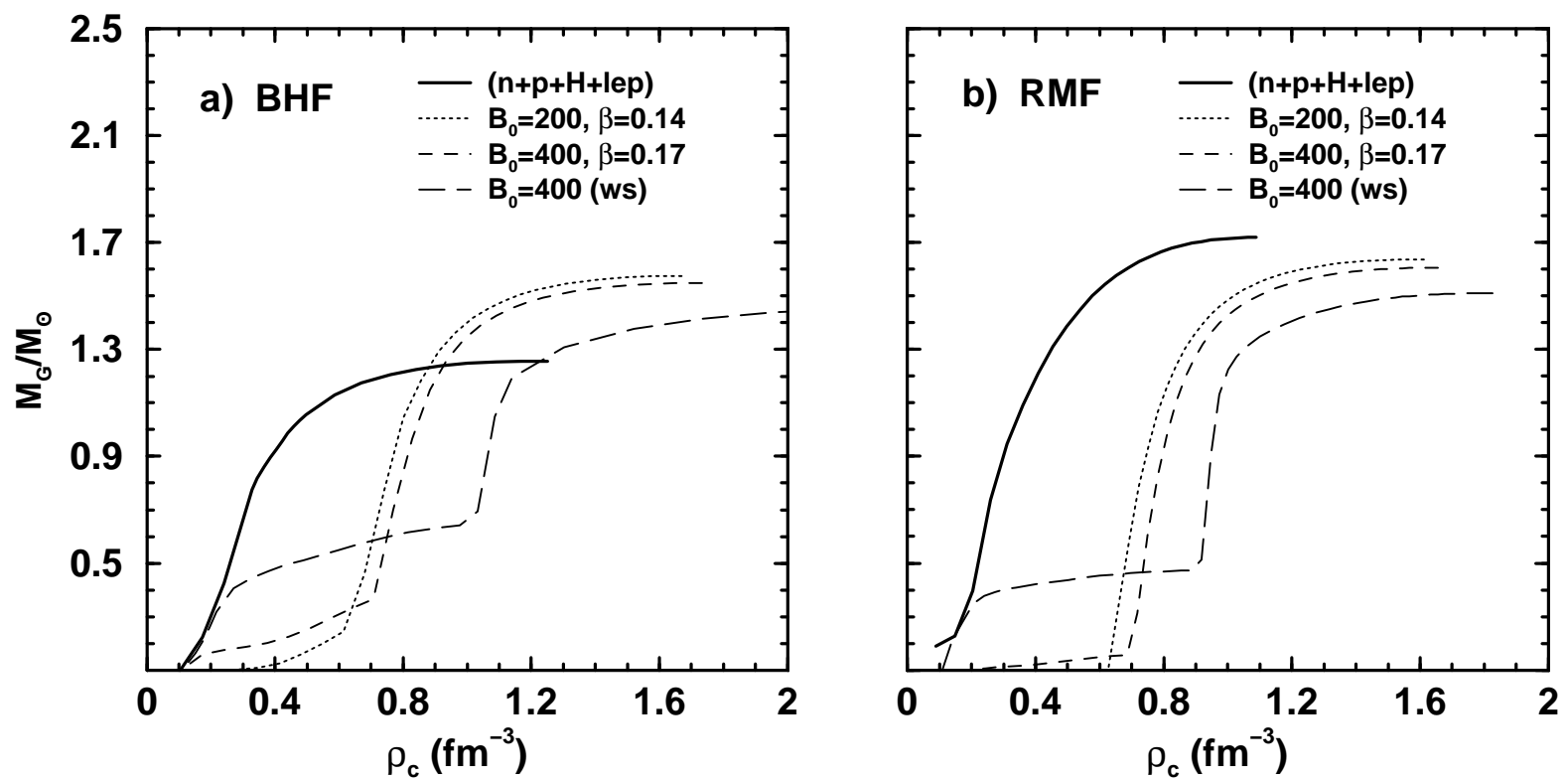


Fig.4

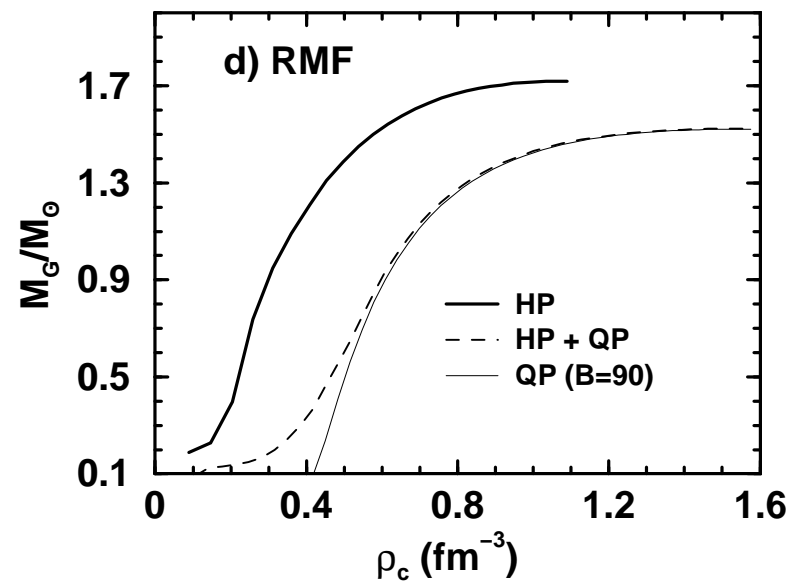
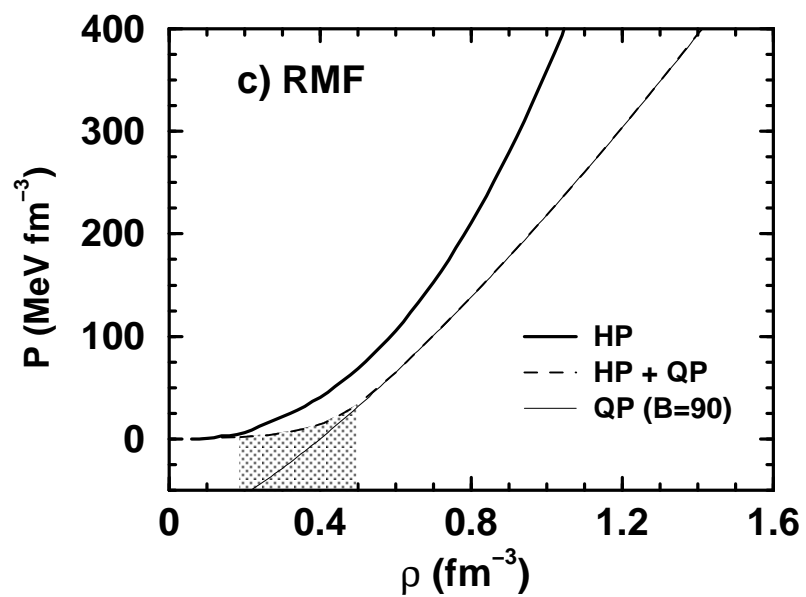
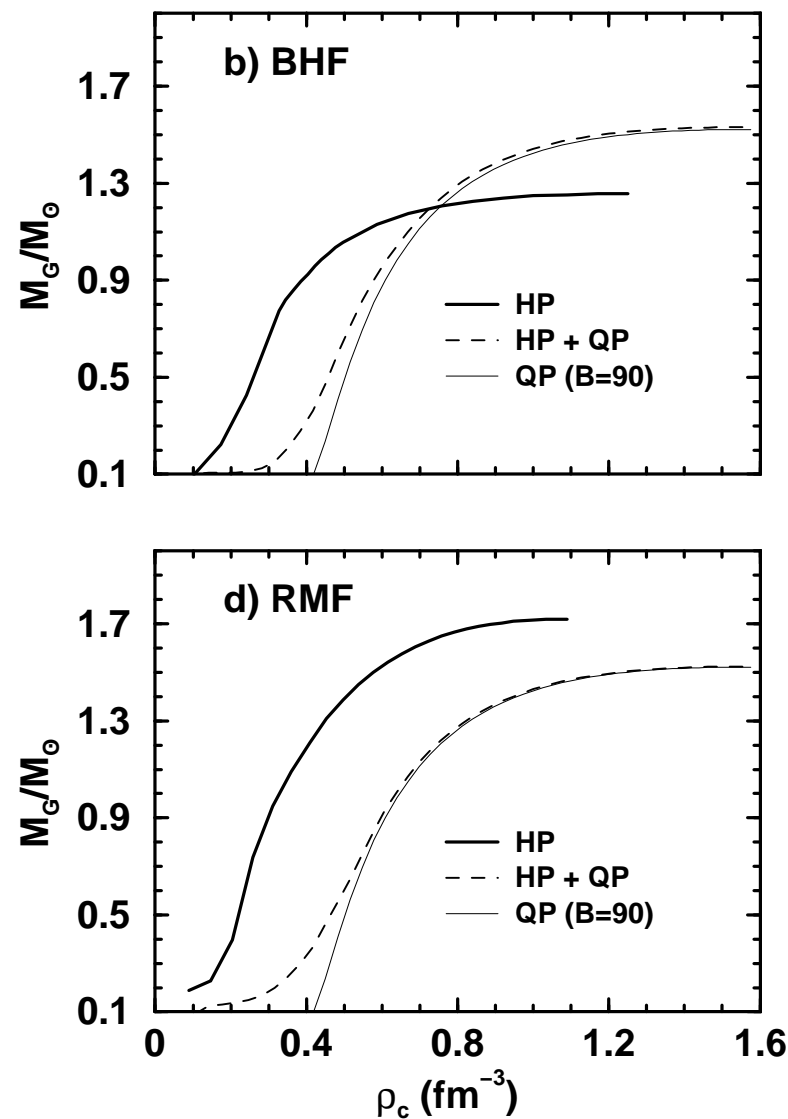
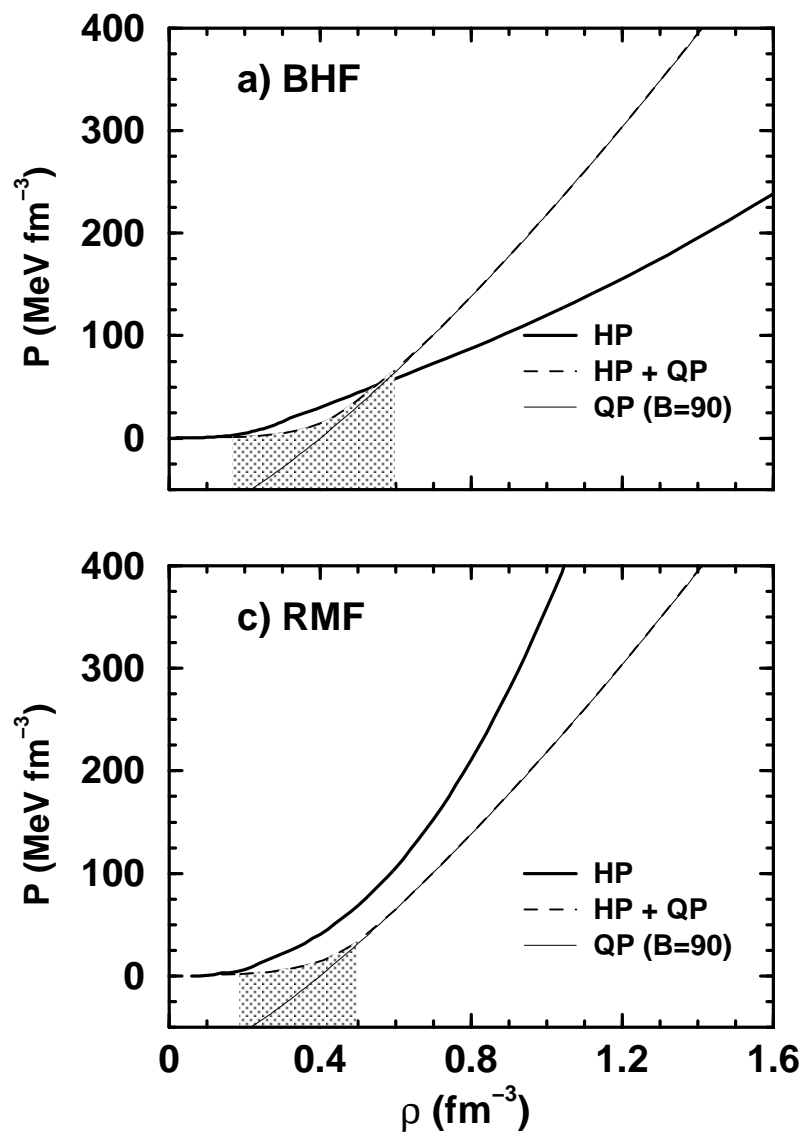


Fig.5